

Reducing Latency in MEC Networks with Short-packet Communications

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Abstract—Different from the conventional transmit-then-compute scheme, which has been widely considered in mobile edge computing (MEC) networks, we propose a transmit-while-compute scheme to reduce the average latency. Specifically, by means of the partial offloading technique, the computational task is divided into multiple subtasks which are sequentially offloaded to the edge server node (ESN) with short-packet communications. Therefore, the transmission and computation of the task can be executed almost simultaneously. Moreover, to minimize the average latency, we first prove the optimization of corresponding tasks is quasi-convex, then provide an optimal solution to determine the blocklength and the size of subtasks. A low-complexity solution based on the alternating optimization (AO) method is proposed as well. Numerical results show that the average latency of the proposed scheme is very close to the computing time of the task.

Index Terms—Mobile edge computing (MEC), short-packet communications.

I. INTRODUCTION

Mobile edge computing (MEC) enables low-cost mobile devices to implement high-performance applications through offloading computational tasks to the edge server nodes (ESNs) [1]. To support real-time intelligent applications in ESN-enabled Internet of Things (IoT) networks, such as tactile Internet and virtual reality, the execution latency of the task is required to be extremely low [1], [2]. To address this issue, short-packet communication is considered, which utilizes finite-blocklength coding and reduces the transmission time of the task to milliseconds or less [3]–[5].

To exploit the benefit of short-packet communication for the MEC network, plenty of works have devoted to the design of offloading strategy [6]–[8]. Specifically, She *et al.* considered the task queuing delay and optimized the computational resource to minimize the packet error rate (PER) in [6]. In [7], Li *et al.* analyzed the average age of information in short-packet MEC networks, and jointly optimized the task offloading ratio and blocklength. In [8], Zhu *et al.* proposed to leverage retransmission in ultra-reliable scenarios and improved the

energy efficiency by adjusting the blocklength. However, these works are based on the conventional transmit-then-compute scheme, and the average latency of the system comprises the transmission and computation time, respectively.

In this paper, we develop a transmit-while-compute scheme for short-packet MEC networks to reduce the average latency. Specifically, the mobile user splits the computational task into multiple subtasks and sequentially offloads the subtasks to the ESN with short-packet communications. Moreover, the average latency minimization problem is solved by the proposed optimal solution and low-complexity alternating optimization (AO) based solution. Numerical results demonstrate the outperformance of the proposed mechanism compared with the benchmark conventional mechanism. In addition, the average latency of the proposed scheme is shown to be close to the computing time of the task.

II. SYSTEM MODEL

In this paper, we investigate a short-packet MEC network, where a mobile device implements an application task with the total workload of N_{tol} bits, while the task is offloaded to the ESN due to the limited computational capability at the mobile device. Based on the data-partition model and partial offloading technique in [1], we propose a transmit-while-compute scheme and lower the average latency of the system. The detailed procedure is illustrated as follows.

A. Communication and computation model

Utilizing the partial offloading technique [1], the computational task is equally divided into $K \in \mathcal{N}$ subtasks with a workload of $N = N_{\text{tol}}/K$ bits, where \mathcal{N} denotes the set of all natural numbers. The subtasks are sequentially offloaded to the ESN with short-packet communication, where each packet contains m channel uses. For each channel use, the time duration is $t = 1/W$ seconds, where W is the allocated bandwidth. As the decoding error occurs in short-packet communications, retransmission is employed to guarantee the reliability of communications. Hence, given maximal transmit power P_S , channel gain $|h|^2$, noise power σ^2 , and received signal-to-noise ratio (SNR) $\gamma = P_S|h|^2/\sigma^2$, the average transmission time for each subtask is

$$T_S(K, m) = \frac{mt}{\bar{\epsilon}}, \quad (1)$$

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where $\bar{\epsilon} = 1 - \epsilon$, and $\epsilon = Q(d(K, m))$ is the PER; $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$ is the Gaussian Q function, and

$$d(K, m) = (Cm^{1/2} - N_{\text{tol}}K^{-1}m^{-1/2})V^{-1}, \quad (2)$$

$$C = \log_2(1 + \gamma), \quad V = \log_2 e(1 - (1 + \gamma)^{-2})^{1/2}. \quad (3)$$

When the transmission of a subtask is completed, the ESN can start to compute the subtask. We assume η central processing unit (CPU) cycles are consumed for each bit of workload, and the computing capability at ESN is f CPU cycles per second. As such, the computing time of each subtask is

$$T_E(K) = \frac{N_{\text{tol}}\eta}{Kf}. \quad (4)$$

B. Transmit-while-compute scheme

To further exploit the benefit of partial offloading, we propose a transmit-while-compute scheme, i.e., the computation of a subtask and the transmission of sequent subtasks can be executed concurrently, which significantly reduces the average latency of the system. Moreover, we analyze the average latency of the system by comparing the values of mt and $T_E(K)$ in the following two cases.

Case 1: When $mt \geq T_E(K)$, the transmission of k th subtask and computing of $(k-1)$ th subtask can be conducted simultaneously. Therefore, the latency is contributed by the transmission of all subtasks and the computing of the last subtask. Hence the average latency of the system $\bar{T}(K, m)$ can be expressed as

$$\begin{aligned} \bar{T}(K, m) &= \bar{T}_1(K, m) \\ &= KT_S(K, m) + T_E(K) = \frac{Kmt}{\bar{\epsilon}} + \frac{N_{\text{tol}}\eta}{Kf}. \end{aligned} \quad (5)$$

Case 2: When $mt < T_E(K)$, the accurate expression of the average latency of the system, i.e., $\bar{T}(K, m)$, is difficult to determine. However, we know that after the computing of the k th subtask, for $k > 1$, the maximal average accumulated latency for the k th subtask is $T_S + T_E(K) - mt$ seconds, which relates to the scenario that the computing time of subtask is always less than the transmission time. This could happen when the PER is large. Therefore, we can derive

$$\begin{aligned} \bar{T}(K, m) &\leq \bar{T}_2(K, m) \\ &= T_S(K) + (K-1)(T_S + T_E(K) - mt) + T_E(K) \\ &= \frac{Kmt}{\bar{\epsilon}} + \frac{N_{\text{tol}}\eta}{f} - (K-1)mt. \end{aligned} \quad (6)$$

where $\bar{T}_2(K, m)$ is the upper bound of the average latency of the system. Moreover, it is clear to see that $\bar{T}_1(K, m) > \bar{T}_2(K, m)$ holds with $mt > T_E(K)$, and $\bar{T}_1(K, m) < \bar{T}_2(K, m)$ holds with $mt < T_E(K)$. Hence, we provide a tight upper bound for the average latency of the system as follows

$$\bar{T}(K, m) \leq \bar{T}_U(K, m) = \max(\bar{T}_1(K, m), \bar{T}_2(K, m)), \quad (7)$$

for $m \in \mathcal{N}$, where the equality sign holds when $K = 1$ or $mt \geq T_E(K)$.

C. Transmit-then-compute scheme

As illustrated in the conventional transmit-then-compute scheme [8], the ESN starts to compute the task after offloading of the whole task. We see that the average latency in (5) with $K = 1$ corresponds to the conventional scheme.

III. PROBLEM FORMULATION AND SOLUTIONS

We aim to minimize the upper bound of average latency of the system $\bar{T}_U(K, m)$ by choosing proper values of m and K , while the optimization problem can be formulated as

$$\min_{m, K} \bar{T}_U(K, m) \quad (8a)$$

$$\text{s.t. } Q(d(K, m)) \leq \epsilon_{\text{max}}, \quad (8b)$$

$$K \leq K_{\text{max}}, \quad (8c)$$

$$m, K \in \mathcal{N}, \quad (8d)$$

where $\epsilon_{\text{max}} < 0.5$ is the maximum PER to ensure the transmission efficiency. In the following, we devise an optimal solution and an AO-based solution to Problem (8).

A. Proposed optimal solution

In this subsection, we propose an optimal solution to Problem (8) based on the one-dimension search and bisection search. By the one-dimension search of K , we can find the optimal m by solving problem

$$\min_m \bar{T}_U(K, m) \quad \text{s.t. } (8b), (8d), \quad (9)$$

where (8b) is equivalent to $m > \bar{m}$, and \bar{m} is a unique point that makes equality sign in (8b) valid [4]. To solve Problem (9), we provide the following lemmas.

Lemma 1: $\bar{T}_1(K, m)$ is a quasi-convex function with respect to (w.r.t.) m for $m \geq \bar{m}$, and there is only one unique minimum of $\bar{T}_1(K, m)$.

Proof 1: See Appendix A.

Lemma 2: $\bar{T}_2(K, m)$ is a quasi-convex function w.r.t. m for $m \geq \bar{m}$ and the minimum of $\bar{T}_2(K, m)$ is unique.

Proof 2: See Appendix B.

Recalling (7), $\bar{T}_U(K, m)$ is the point-wise maximum of $\bar{T}_1(K, m)$ and $\bar{T}_2(K, m)$, which are both quasi-convex functions from Lemma 1-2. From [9], $\bar{T}_U(K, m)$ is a quasi-convex function w.r.t. m , and we can obtain the optimal $m^*(K)$ to Problem (9) by applying the bisection method. Through the one-dimension search and bisection method, we can obtain the optimal K and m for Problem (8) as follows

$$(m^*, K^*) = \arg \min_{K \in [1, K_{\text{max}}]} \bar{T}_U(K, m^*(K)). \quad (10)$$

B. Alternating optimization based solution

The one-dimension search is complicated with a large number of K_{max} . To lower the computational complexity, we leverage the AO method to obtain K and m iteratively. In detail, given K , we can solve Problem (11) and obtain the corresponding optimal $m^*(K)$ from the above subsection. Given m , the optimal K can be obtained by solving the sequent problem

$$\min_K \bar{T}_U(K, m) \quad \text{s.t. } (8b) - (8d), \quad (11)$$

where (8b) is equivalent to $K > \bar{K}$, and \bar{K} is a unique point that makes the equality sign in (8b) valid. To solve Problem (11), we provide the following lemma.

Lemma 3: $\bar{T}_U(K, m)$ is a convex function w.r.t. $K > \bar{K}$.

Proof 3: See Appendix C.

Based on Lemma 3, we can obtain the optimal $K^*(m)$ to Problem (14) via the bisection method.

IV. NUMERICAL RESULTS

In this section, we present the experimental results of the proposed scheme and solutions to validate their effectiveness.

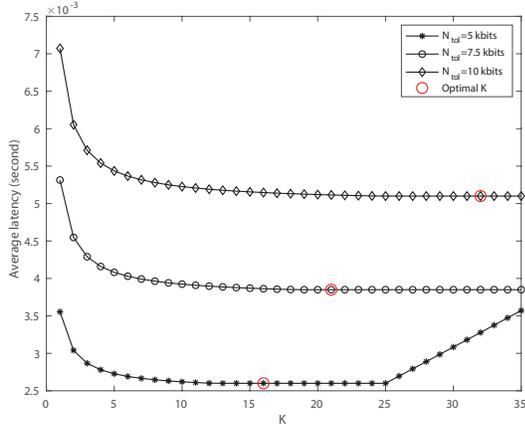


Fig. 1. Average latency versus number of subtasks K .

Without loss of generality, we set $f = 1$ GHz, $\eta = 500$. The allocated bandwidth W is 1 MHz, then we have $t = 10^{-6}$ seconds. In addition, the received SNR at ESN is $\gamma = 15$ dB and the maximum PER is $\epsilon_{\max} = 10^{-9}$. Also, we set $K_{\max} = 100$.

In Fig. 1, we show the minimal average latency with different values of K . For different numbers of N_{tol} , the optimal K is highlighted with a red circle. Compared with the conventional scheme with $K = 1$, where the detailed process can be found in [8], the proposed scheme can significantly reduce the average latency. In Fig. 2, the convergence behavior

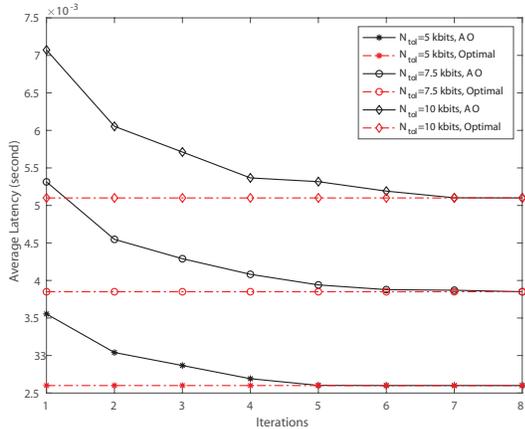


Fig. 2. Average latency versus number of iterations.

of the proposed AO solution is provided, and the results of the optimal solution are given for comparison, where $N_{\text{tol}} \in \{5, 7.5, 10\}$ Kbits. As shown in Fig. 2, the results of AO-based solution converge with several iterations, and they are close to the optimal results.

In Fig. 3, the variations of the average latency with the total workload N_{tol} are demonstrated, where N_{tol} varies from 5 to 50 Kbits and $\eta \in \{500, 1000\}$. We denote the conventional scheme and the proposed scheme as “ $K=1$ ” and “Optimal K ”, respectively. The computing time of the whole task is provided as a benchmark, which is the minimum achievable latency. As we can see, the gap between the results of the conventional scheme and the proposed scheme increases. Moreover, the

results of the proposed scheme are very close to the computing time of the whole task, since the transmission and computation of subtasks are done almost simultaneously.

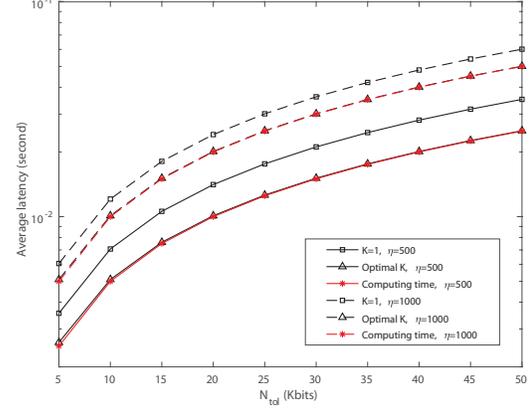


Fig. 3. Average latency versus total workload.

V. CONCLUSION

In this paper, we propose a transmit-while-compute scheme with short-packet communications to reduce the average latency of MEC networks. Moreover, the average latency minimization problem is solved with the proposed optimal solution and the low-complexity AO-based solution. Numerical results validate the effectiveness of the proposed scheme and solutions.

APPENDIX A

PROOF OF LEMMA 1

From (5), Lemma 1 can be proved by showing that $T_S(K, m)$ is a quasi-convex function w.r.t. m . To this end, we first derive the second-order derivative of $\bar{\epsilon}$ w.r.t. m , i.e.,

$$\begin{aligned} \frac{\partial^2 \bar{\epsilon}}{\partial m^2} &= Q''(-d(K, m)) \left(\frac{\partial d(K, m)}{\partial m} \right)^2 \\ &+ Q'(-d(N, m)) \frac{\partial^2 d(K, m)}{\partial m^2}, \end{aligned} \quad (12)$$

where

$$\frac{\partial d(K, m)}{\partial m} = \frac{Cm^{-1/2} + Nm^{-3/2}}{2V} > 0, \quad (13)$$

$$\frac{\partial^2 d(K, m)}{\partial m^2} = -\frac{Cm^{-3/2} + 3Nm^{-5/2}}{4V} < 0, \quad (14)$$

From [4], when $Q(-x) > 1/2$ holds, we have

$$Q'(-x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} > 0, \quad Q''(-x) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} < 0, \quad (15)$$

which suggests that the value of (12) is negative and $\bar{\epsilon}$ is a strictly concave function w.r.t. m . From (5), $T_S(K, m)$ is the composition of a linear function divided by a concave function, which is a quasi-convex function w.r.t. m and also relates to only one minimum [9].

APPENDIX B

PROOF OF LEMMA 2

Take the first-order derivative of $\bar{T}_2(K, m)$ in (6) w.r.t. m to 0, and the roots to that, i.e., \tilde{m} should satisfy

$$\left(\bar{\epsilon} - \frac{\partial \bar{\epsilon}}{\partial m} m - \bar{\epsilon}^2 \frac{K-1}{K} \right) \Big|_{m=\tilde{m}} = 0, \quad (16)$$

where $\frac{\partial \bar{\epsilon}}{\partial m} = Q'(-d(K, m)) \frac{\partial d(K, m)}{\partial m}$.

To analyze the convexity of $\bar{T}_2(K, m)$, we compute the second-order derivative of $\bar{T}_{2,U}(K, m)$ w.r.t. m and derive

$$\frac{\partial^2 \bar{T}_2(K, m)}{\partial m^2} = \frac{Kt\lambda}{\bar{\epsilon}^4}, \quad (17)$$

where

$$\lambda = -\bar{\epsilon}^2 \frac{\partial^2 \bar{\epsilon}}{\partial m^2} m - 2\bar{\epsilon}^2 \frac{\partial \bar{\epsilon}}{\partial m} + \bar{\epsilon} \left(\frac{\partial \bar{\epsilon}}{\partial m} \right)^2 m. \quad (18)$$

From the sufficient condition of quasi-convex function in [9], $\bar{T}_2(K, m)$ is a quasi-convex function w.r.t. m if the value in (17) is positive, which is equivalent to $\lambda < 0$, when $m = \tilde{m}$. When $m = \tilde{m}$, substituting (16) into (18), λ can be written as

$$\lambda = -\delta(\tilde{m})\bar{\epsilon}^2 \Big|_{m=\tilde{m}}, \quad (19)$$

where $\delta(m)$ is defined as

$$\delta(m) = \frac{\partial^2 \bar{\epsilon}}{\partial m^2} m + 2 \frac{\partial \bar{\epsilon}}{\partial m} \bar{\epsilon} \frac{K-1}{K}. \quad (20)$$

Also, from (12), we find that $\delta(\tilde{m}) < 0$ holds when $K = 1$.

When $K \geq 2$, we have $\delta(\tilde{m}) < 0$ if $R(\tilde{m}) \geq 1$, where

$$\begin{aligned} R(m) &= \left(-\frac{\partial^2 \bar{\epsilon}}{\partial m^2} m \right) / \left(2 \frac{\partial \bar{\epsilon}}{\partial m} \bar{\epsilon} \right) \\ &= \frac{C^2 m - N^2 m^{-1}}{4V^2 \bar{\epsilon}} + \frac{Cm^{-1/2} + 3Nm^{-3/2}}{4\bar{\epsilon}(Cm^{-1/2} + Nm^{-3/2})}. \end{aligned} \quad (21)$$

Also, $R(m) = 1$ holds when $\bar{\epsilon} = 1/2$. Therefore, $R(\tilde{m}) \geq 1$ holds if $R'(\tilde{m}) \geq 0$. Compute $R'(m)$, which is given by

$$\begin{aligned} R'(m) &= \frac{(C^2 + (N/m)^2)\bar{\epsilon} - \frac{\partial \bar{\epsilon}}{\partial m}(C^2 m - N^2 m^{-1})}{4V^2 \bar{\epsilon}^2} \\ &\quad - \frac{\frac{\partial \bar{\epsilon}}{\partial m}(C + 3Nm^{-1})}{4\bar{\epsilon}^2(C + Nm^{-1})} - \frac{CNm^{-1}}{2\bar{\epsilon}(Cm^{1/2} + Nm^{-1/2})^2}. \end{aligned} \quad (22)$$

From (16), we have

$$\frac{\partial \bar{\epsilon}}{\partial m} = \frac{\bar{\epsilon} - \bar{\epsilon}^2(K-1)/K}{m} < \frac{3\bar{\epsilon}}{4m}, \quad (23)$$

when $m = \tilde{m}$ and $\bar{\epsilon} > 0.5$. Using (23) and the inequality of arithmetic and geometric means, we can find that

$$R'(m) > \left(\frac{C^2 + 7(N/\tilde{m})^2}{V^2} - \frac{11}{\tilde{m}} \right) \frac{1}{16\bar{\epsilon}} \Big|_{m=\tilde{m}}, \quad (24)$$

Following (24), we see that $R'(\tilde{m}) > 0$ holds if

$$\tilde{m}^2 C^2 - 11\tilde{m}V^2 + 7N^2 \geq 0. \quad (25)$$

Based on the discriminant of the quadratic inequality, if

$$121V^4 - 28(CN)^2 \leq 0, \quad (26)$$

then (25) holds, and (26) can be rearranged into

$$N \geq 11V^2(2\sqrt{7}C)^{-1}. \quad (27)$$

Computing the first-order derivative of V^2/C w.r.t. γ , we can evaluate the maximum of V^2/C and find that $11V^2(2\sqrt{7}C)^{-1}$ is smaller than $5/2$. Taking the fact that the workload of a subtask, i.e., $N = N_{\text{tot}}/K$, is generally greater than several bits in practice, we can conclude that both $\delta(\tilde{m})$ and λ are negative with $m = \tilde{m}$. Hence $\bar{T}_2(K, m)$ is a quasi-convex function w.r.t. m . If \tilde{m} does not exist, $\bar{T}_2(K, m)$ is a monotone function w.r.t. m and still a quasi-convex function.

In addition, if the minimum of $\bar{T}_2(K, m)$ is not unique, then there is at least one \tilde{m} that makes the value of second-order derivative of $\bar{T}_2(K, m)$ w.r.t. m in (17) negative with $m = \tilde{m}$, which contradicts to the results in (21)-(27). The proof of Lemma 2 is completed.

APPENDIX C PROOF OF LEMMA 3

We first rewrite $\bar{T}_U(K, m)$ as

$$\bar{T}_U(K, m) = KT_S(K, m) + \phi(K), \quad (28)$$

where $\phi(K) = \max(T_E(K), N_{\text{tot}}\eta f^{-1} - (K-1)mt)$.

To continue, we calculate the second-order derivative of $KT_S(K, m)$ w.r.t. K as

$$\frac{\partial^2 KT_S(K, m)}{\partial K^2} = \frac{\psi(K)}{\bar{\epsilon}^3}, \quad (29)$$

where

$$\psi(K) = -\bar{\epsilon} \frac{\partial^2 \bar{\epsilon}}{\partial K^2} - 2\bar{\epsilon} \frac{\partial \bar{\epsilon}}{\partial K} + 2K \left(\frac{\partial \bar{\epsilon}}{\partial K} \right)^2. \quad (30)$$

From (1), we can compute

$$\frac{\partial \bar{\epsilon}}{\partial K} = Q'(-d(K, m)) \frac{\partial d(K, m)}{\partial K} < 0, \quad (31)$$

since $Q'(-d(K, m)) > 0$ from (15) and

$$\frac{\partial d(K, m)}{\partial K} = -\frac{N_{\text{tot}}m^{-3/2}}{2VK^2} < 0. \quad (32)$$

Similarly, we can derive

$$\begin{aligned} \frac{\partial^2 \bar{\epsilon}}{\partial K^2} &= Q''(-d(K, m)) \left(\frac{\partial d(K, m)}{\partial K} \right)^2 \\ &\quad + Q'(-d(K, m)) \frac{\partial^2 d(K, m)}{\partial K^2} < 0, \end{aligned} \quad (33)$$

since $Q''(-d(K, m)) < 0$ from (15) and

$$\frac{\partial^2 d(K, m)}{\partial K^2} = \frac{N_{\text{tot}}m^{-3/2}}{VK^3} > 0. \quad (34)$$

From (30)-(34), we know that $\psi(K) > 0$ and $KT_S(K, m)$ is a convex function w.r.t. K . Moreover, $\phi(K)$ is the point-wise maximum of two convex functions, thus is convex. As such, $\bar{T}_U(K, m)$ is the sum of two convex functions $KT_S(K, m)$ and $\phi(K)$, hence a convex function w.r.t. K .

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